(Pages : 2) **W – 5616**

Reg. No. :

Name :

First Semester M.Sc. Degree Examination, January 2026

Statistics

ST 513 – PROBABILITY THEORY – I

(2021 Admission Onwards)

Time : 3 Hours Max. Marks :60

PART – A

Answer any **four** questions. Each carries **3** marks.

1 Define limit supremum and limit infimum of a sequence of sets.

2. Define Caratheodoery extension theorem.

3 State the monotone convergence theorem.

4. What do you mean by absolute continuity and singularity of measures?

5. Define conditional probability measure.

6 State Bayes theorem.

7 Define random variable If X and Y are random variables then prove that $X+Y$ is also a random variable.

8. What do you mean by induced probability space?

(4 × 3 = 12 Marks)

PART – B

Answer any **three** question. Each question carries **8** marks.

9 Show that the intersection of arbitrary number of fields is a field. Also show that the union of two fields may not be a field.

10. Define converges in p^{th} mean. State and prove continuity property of measure.

11 Define the following

(a) Minimal sigma field,

(b) Generated sigma field, and

(c) Induced sigma field.

P.T.O.

12. State and prove Fatou's theorem.

13. Show that if two functions are integrable. then their linear combination is also integrable.

14. State Radon Nykodym theorem and give any one of its applications.

(3 × 8 = 24 Marks)

PART – C

Answer any **three** question. Each question carries **8** marks.

15. State and prove Kolmogorov 0-1 law.

16. If $\{A_n, n \geq 1\}$ is a sequence of independent events and if $\sum_{n=1}^{\infty} P(A_n) = \infty$, then show that $P(\overline{\lim}(A_n)) = 1$.

17. Explain the monotone and continuity properties of probability measure.

18. Define vector random variable. Also explain the sigma field induced by a sequence of random variables

19. Define distribution function. State and prove the properties of distribution function.

20. State and prove the Jordan decomposition theorem.

(3 × 8 = 24 Marks)

(Pages : 2) **U – 6595**

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, February 2025

Statistics

ST 513 : PROBABILITY THEORY — I

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions. **Each** carries **3** marks.

1. Define limit supremum and limit infimum of a sequence of sets. If $\{A_n\}$ is an increasing sequence of sets what are the limit supremum and limit infimum?

2. Define minimal sigma field and generated sigma field.

3. State and prove Fatou's theorem.

4. Define absolute continuity and singularity of measures.

5. Describe the monotone property of probability measure.

6. State the Borel-Cantelli lemma.

7. What do you mean by induced probability measure?

8. Define the concept of mixture of distribution functions.

(4 × 3 = 12 Marks)

PART – B

Answer **any three** questions. **Each** carries **8** marks.

9. Distinguish between a field and a σ- field. Let $\Omega = \{1, 2, 3, ...\}$ and C be the class of subsets A of Ω such that either A contains a finite number of points or A^c contains a finite number of points. Examine whether C is field or a σ- field or both.

10. Define Outer measure, Lebesgue measure and Lebesgue-Stieltjes measure.

11. Prove that a sequence $\{f_n\}$ of almost everywhere finite measurable functions converges almost everywhere to a finite measurable function f if and only if the sequence mutually converges almost everywhere

12. State and prove Lebesgue dominated convergence theorem.

13. If f and g are measurable integrable functions then show that

$$\int (f+g)d\mu = \int f\, d\mu + \int g\, d\mu.$$

14. State the Radon-Nykodym theorem and give its application.

(3 × 8 = 24 Marks)

PART – C

Answer **any three** questions. **Each** carries **8** marks.

15. State and prove Kolmogrov 0-1 law.

16. Define the mutual independence and pair wise independence of events. Prove that mutual independence implies pair wise independence but the converse need not be true.

17. State and prove the Bayes Theorem.

18. State and prove Jordan decomposition theorem for distribution functions.

19. Define distribution function. State and prove the properties of distribution function.

20. Let X and Y are independent random variables with distribution functions F and G respectively. Find the distribution function of $X+Y$ and XY in terms of F and G.

(3 × 8 = 24 Marks)

(Pages : 2) **S – 6273**

Reg. No. :

Name :

First Semester M.Sc. Degree Examination, April 2024

Statistics

ST 513 : PROBABILITY THEORY – I

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any** four questions. Each question carries **3** marks.

1. Define additive set functions and sigma additive set functions.

2. Define Finite measure space and sigma finite measure space.

3. State the Lebesgue dominated convergence theorem.

4. What do you mean by singularity of measures?

5. Define probability measure and probability space.

6. State Borel–Cantelli lemma.

7. What do you mean by mixture of distribution functions?

8. Define random vectors and distribution function of random vectors.

(4 × 3 = 12 Marks)

P.T.O.

PART – B

Answer any **three** questions. Each question carries **8** marks.

9. Show that every function X on Ω is measurable with respect to the power set of Ω.

10. Show that a sigma field is a monotone field and conversely.

11. Define the following: (a) Point-wise convergence, (b) Almost everywhere convergence. (c) Uniform convergence, and (d) Convergence in measure.

12. Define Lebesgue integral and give its properties.

13. State and prove monotone convergence theorem.

14. Define Lebesgue–Stieltjes integral and give the conditions under which it reduces to Riemann integral.

(3 × 8 = 24 Marks)

PART – C

Answer any **three** questions. Each question carries **8** marks.

15. State and prove Bayes theorem.

16. State and prove Borel 0–1 law

17. Describe the continuity property of a probability measure.

18. State and prove Jordan decomposition theorem for distribution functions.

19. Define distribution function. Show that a distribution function has atmost a countable number of discontinuities.

20. Let $\{X_n\}$ be independently uniformly distributed on $(0,1)$. Show that $Y_n = \max_{1\le k\le n}(X_k) \to 1$ as $n \to \infty$ almost surely.

(3 × 8 = 24 Marks)

(Pages : 2) **R – 6198**

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, May 2023

Statistics

ST 513 – PROBABILITY THEORY - I

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 60

PART – A

Answer any **four** questions. Each question carries **3** mark.

1. Prove that a monotone sequence of sets converges to a set.
2. Does point-wise convergence imply almost everywhere?
3. Define Lebesgue measure. Mention the important properties of Lebesgue measure.
4. State the Lebesgue dominated convergence theorem.
5. Define probability as a measure.
6. Establish the continuity property of probability.
7. State the properties of the cumulative distribution function of a random variable.
8. Define a mixture distribution. If $F_x(x) = 1 - p + p(1 - e^{-\theta x})$, identify the discrete part and continuous part of the decomposition.

(4 × 3 = 12 Marks)

P.T.O.

PART – B

Answer any **three** questions. Each question carries **8** marks.

9. Define a monotone field. Say true or false: "Every σ field is a monotone field and conversely". Justify your answer.

10. Prove that a Borel field $\mathfrak{B}$ on R is the minimal σ field generated by any finite interval on R where the members of $\mathfrak{B}$ are Borel sets or intervals on R.

11. Give the general definition of integral of a non-negative simple measurable function, integral of a non-negative measurable function and integral of a measurable function. Also show that the non-negative measurable function f is integrable $\leftrightarrow |f|$ is integrable.

12. State and prove the Fatou's Theorem.

13. Discuss on Riemann-Stieltjes integral and Lebesgue-Stieltjes integral and their properties. Establish when Lebesgue-Stieltjes integral reduces to the Riemann-Stieltjes integral and also to Riemann integral.

14. Define absolute continuity of a measure with respect to another measure. Discuss its use in finding the Radon-Nikodym derivative of a Borel function. Also mention some properties and applications of Radon-Nikodym derivative.

(3 $\times$ 8 = 24 Marks)

PART – C

Answer any **three** questions. Each question carries **8** marks.

15. State and prove Borel-Cantelli Lemma. Mention its applications.

16. Define conditional probability. State and prove the Bayes Theorem.

17. Define independence of three or more events. Prove or disprove: Mutual independence of events $\leftrightarrow$ Pairwise independence of them.

18. Show that Borel functions of independent r.vs are independent.

19. Show that inverse images preserve all set operations.

20. Define the discontinuities of a distribution function. Also show that the set of discontinuity points of a distribution function is atmost countable.

(3 $\times$ 8 = 24 Marks)

(Pages : 3) **N – 5388**

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, May 2022

Statistics

ST 513 – PROBABILITY THEORY – I

(2021 Admission)

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions. **Each** question carries **3** marks.

1. Say true or false: Every sigma field contains the trivial sigma field. Establish your claim.

2. State Caratheodory extension theorem.

3. Show that inverse image of a minimal σ field is the minimal σ field over the inverse image.

4. Define Lebesgue-Steltjes integral. When does it becomes the Rieman-Stieltjes integral?

5. Examine whether $\overline{\lim}\, A_n$ exists for the sequence of sets $\{A_n; n \geq 1\}$ where $A_n = \left\{x / 0 < x < b + \frac{(-1)^n}{n}\right\}$, where $b > 1; n = 1,2,....$ If so, find it.

6. State the Kolmogrov 0-1 law.

7. $X : \Omega \rightarrow R$ is a random variable if and only if $X^{-1}(-\infty, x] = [X \leq x] \in \mathfrak{B}\, \forall x \in R,$ where is the Borel field on R. Prove.

8. Show that the indicator function I_A is a random variable.

(4 × 3 = 12 Marks)

P.T.O.

PART – B

Answer **any three** questions. **Each** question carries **8** marks.

9. Define $\underline{\lim}, \overline{\lim}$ and limit of a sequence of sets. Show that every monotone sequence of sets is convergent.

10. Is the union and intersection of an arbitrary number of fields (or σ fields) a field (or σ field)? Justify your answer.

11. Discuss in detail the various modes of convergence and their mutual implications of sequence of measurable functions like point-wise convergence, almost everywhere convergence, uniform convergence and convergence in measure.

12. State and prove the Radon-Nikodym theorem and mention any one application of it.

13. Distinguish between Lebesgue measure and Lebesgue – Stieltjes measure. Mention the important properties of both. How can you view Lebesgue – Stieltjes measure as a probability measure?

14. State and prove the Lebesgue dominated convergence theorem.

(3×8 = 24 Marks)

PART – C

Answer **any three** questions. **Each** question carries **8** marks.

15. Define a probability measure. State and prove the monotone properties of a probability measure.

16. State and prove the Borel 0-1 law. When does its converse true?

17. Define independence of random variables. If $f(x,y)=\frac{1}{4}(1+xy), |x|<1, |y|<1$, then show that X^2 and Y^2 are independent but X and Y are not.

18. Show that the σ-field induced by a sequence of random variables is the minimal the σ-field containing the inverse images of all the random variables in the sequence.

19. Give the definitions of discrete and continuous distribution functions. What is the concept of a general distribution function and a mixture of distribution functions?

20. State and prove the Hann-Jordan decomposition theorem.

($3 \times 8 = 24$ Marks)
