(Pages : 3)

V – 5921

Reg. No. :

Name : ...

Second Semester M.Sc. Degree Examination, August 2025

Statistics

ST 522 : DISTRIBUTION THEORY

(2021 Admission Onwards)

Time : 3 Hours

Max. Marks : 60

SECTION – A

Answer **any four** questions. **Each** question carries **3** marks.

1. Define probability generating function of a random variable. For what type of random variables it is defined? Justify its name.
2. Show that a left truncated Geometric r.v is again Geometric.
3. Define a degenerate r.v and hence write its distribution function.
4. Define the Generalized Power Series distribution? Show that the Negative Binomial is a special case of it.
5. Obtain the distribution of X^c if X has a Weibull distribution with shape parameter, $c > 0$.
6. Show that a Beta (α, β) distribution belongs to the two parameter exponential family of distributions.
7. If $X_1, X_2, \ldots\ldots X_n$ are iid random variables having uniform distribution in $(0, \theta)$, obtain the marginal p.d.f of $X_{(1)}$ and $X(n)$.
8. Define a non-central t statistics, mentioning its parameters.

(4 × 3 = 12 Marks)

P.T.O.

SECTION – B

Answer **any three** questions. **Each** question carries **8** marks.

9. Derive the PGF of the Negative Binomial distribution and hence its k^{th} descending factorial moments and there by obtain its mean and variance.

10. Show that $\mu_{X,Y}^2 \leq \sigma_X^2 \sigma_Y^2$, where $\mu_{X,Y}^2$ is the bivariate central moment of order 2 of the random vector (X, Y).

11. Define a mixture distribution. Show that $F(x) = (1-p) + p(1-e^{-\theta x})$ is a mixture of distribution functions.

12. Show that a necessary and sufficient condition for the Power Series Distribution to be Poisson is that mean = variance.

13. Explain the capture-recapture problem and the use of Hyper geometric distribution to estimate the population of a finite dichotomous population.

14. Define a multinomial distribution. For a multinomial distribution with usual notations, show that $Corr(X_i, X_j) = -\sqrt{\dfrac{P_i P_j}{(1-P_i)(1-P_j)}}$.

(3 × 8 = 24 Marks)

SECTION – C

Answer **any three** questions. **Each** question carries **8** marks.

15. Show that for a lognormal distribution $\dfrac{\text{mode}}{\text{mean}} = \left(\dfrac{\text{median}}{\text{mean}}\right)^3$.

16. Establish that the Pearson System of distributions is completely specified by four equations. Solve for the parameters and hence write the criterion 'K' to identify the members.

17. Define the Bivariate exponential Distribution due to Gumbel. Obtain the marginal distributions of it.

18. If X and Y are independent Rectangular variates on [0, 1], find the distributions of (a) $X+Y$, (b) $X-Y$, (c) $|X-Y|$.

19. Derive the distribution of the Sample Median while a random sample of size n is taken from a population having an absolutely continuous distribution function.

20. If $F(n_1, n_2)$ has an F distribution with (n_1, n_2)d.f, then show that $\frac{1}{F(n_1, n_2)}$ has an $F(n_2, n_1)$ distribution. Deduce that $P[F_\alpha(n_1, n_2) \geq c] = P\left[F_{1-\alpha}(n_2, n_1) \leq \frac{1}{c}\right], c > 0$.

(3 × 8 = 24 Marks)

(Pages : 3) **T – 6379**

Reg. No. : .....

Name : .....

Second Semester M.Sc. Degree Examination, September 2024

Statistics

ST 522: DISTRIBUTION THEORY

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 60

PART – A

Answer any **four** questions. Each question carries **3** marks.

1. Examine whether $P(s) = \frac{2}{1+s}; |s| < 1$, is a probability generating function. Justify.

2. Define a truncated distribution. Write the p.m.f. of a right truncated Binomial distribution.

3. Define the Modified Power Series distribution. Give any one example of it.

4. Generate the Negative Binomial distribution by compounding Poisson with Gamma.

5. Define Logistic Distribution and mention its important applications.

6. Find the distribution function of a Cauchy random variable.

P.T.O.

7. If $X_1, X_2, \cdots, X_n$ are iid random variables having exponential distribution with parameter θ, obtain the distribution of $X_{(1)} = \text{mini}\{X_1, X_2, \ldots, X_n\}$

8. Define a non-Central F statistic mentioning its parameters.

($4 \times 3 = 12$ Marks)

PART – B

Answer any **three** questions. Each question carries **8** marks.

9. If $(X,Y) \sim BVN(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$, show that $X+Y$ and $X-Y$ are independent if and only if $\sigma_x^2 = \sigma_y^2$. Use this result to prove the independence of the sample mean and sample variance.

10. Define characteristic function of a random variable. Why do you say that it exists always? Mention its properties. Prove that a characteristic function is real and even if and only if the distribution function is symmetric.

11. Explain compounding of distributions. Show that the Pareto Type II distribution is generated as a compounding distribution *of* negative exponential with Gamma.

12. Show that X and Y are Geometric random variables, if and only if they are independent and identically distributed with
$P(X = s / X+Y = s) = P(X = s-1 / X+Y = S) = \frac{1}{s+1}$.

13. If $X_1 \sim Binomial(n_1, p)$ and $X_2 \sim Binomial(n_2, p)$ and are independent, then obtain the distribution of $X_1/(X_1+X_2)$.

14. Define factorial moments. Establish the recurrence relation between the descending factorial moments of a Poisson distribution with parameter λ.

($3 \times 8 = 24$ Marks)

PART – C

Answer any **three** questions. Each question carries **8** marks.

15. Obtain the characteristic function of the Laplace (double exponential) distribution. Identify its form with the pdf of a well known distribution.

16. Define a three parameter Weibull distribution. Let X_i, $i = 1,2,..., n$ be i.i.d Weibull random variables with parameters (ξ, α, c), then find the distribution of $mini(X_1, X_2, .., X_n)$.

17. Write the pdf of the Bivariate Normal Distribution. Also derive its conditional densities and show that $V(X) \geq V(X/Y)$

18. Let X_1 follows Gamma (α_1) and X_2 follows Gamma (α_2) and are independent. Find the distribution of

    (a) $\frac{X_1}{X_2}$ and

    (b) $X_1/(X_1 + X_2)$

19. Derive the distribution of the sample Median while a random sample of size n is taken from a continuous $U(0,1)$ population (for both odd and even cases) . Find the mean and variance when n is odd.

20. Derive the pdf of a central χ^2 distribution. Establish the moment generating function and cumulant generating function and hence show that it tends to the Normal distribution as n becomes large.

($3 \times 8 = 24$ Marks)

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(Pages : 3) **R – 7462**

Reg. No. : .....

Name : .....

Second Semester M.Sc. Degree Examination, November 2023

Statistics

ST 522 : DISTRIBUTION THEORY

(2021 – Admission onwards)

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions. **Each** question carries **3** marks.

1. The joint probability density function of two random variables *X and Y* are given $P(X=1, y=-1)=\frac{1}{3}$, $P(X=0,Y=1)=\frac{1}{3}$ *and* $P(X=1,Y=1)=\frac{1}{3}$. Find the marginal distribution of *X* and *Y*.
2. Define truncated distribution. Write the probability mass function of truncated poisson distribution at the point $X=0$.
3. Derive the probability generating function of Binomial distribution. Hence find its mean and variance.
4. Find the marginal distributions *X* and *Y* from trinomial distribution.
5. Define Pareto distribution and mention it's Important characteristics.
6. If $X_1, X_2..., X_n$ are independent random variables having exponential distribution with pdf.

$$f(x;\theta)=\begin{cases}\theta e^{-\theta x}, & x>0,\theta>0\\ 0, & otherwise\end{cases}$$

Obtain the distribution of min $(X_1, X_2,..., X_n)$.

P.T.O.

7. Derive the mode of F distribution.

8. Distinguish between central and Non-central t-distribution.

($4 \times 3 = 12$ Marks)

PART – B

Answer any **three** questions. **Each** question carries **8** marks.

9. Suppose that two dimensional continuous random variables (X,Y) has joint pdf given by $f(x,y)=\begin{cases}6x^2y & ,0<x<1,\ 0<y<1\\ 0, & \text{elsewhere}\end{cases}$ Verify that $\int_0^1\int_0^1 f(x,y)dxdy=1$.

   Find (a) $P(X+Y<1)$, (b) $P(X>Y)$, (c) $P(X<1|Y<2)$

10. Let X and Y be jointly distributed with pdf $f(x,y)=\frac{1}{4}(1+xy), |x|<1, |y|<1$, and 0, otherwise. Show that X and Y are not independent but X^2 and Y^2 are independent.

11. If (X,Y) possess a bivariate normal distribution, find conditional expectation and conditional variance of $X|Y=y$.

12. Let X and Y be independent and identically distributed random variables following geometric distribution with parameter p. Find the conditional probability $P(X=t|X+Y=n)$.

13. Define the Hyper geometric distribution. Show that under conditions the hyper geometric distribution tends to the binomial distribution.

14. Derive the moment generating function of power series distribution. Give any two particular cases of generalized power series distribution (g.p.s.d.).

($3 \times 8 = 24$ Marks)

PART – C

Answer any **three** questions. **Each** question carries **8** marks.

15. Find the characteristic function of Standard Laplace distribution and obtain the cumulants.

16. Derive normal distribution as a special case of Pearson system.

17. If X and Y are independent Gamma variates with parameters α and β respectively. Show that the variables $U = X + Y$ and $V = \frac{x}{x+y}$ are independent. Find the distribution of U and V.

18. If X follows standard normal distribution and Y follows Chi square distribution with n degrees of freedom and both are independent obtain the distribution of $\frac{x}{\sqrt{Y/n}}$.

19. Define Non central Chi-square distribution and derive the pdf. State and prove the additive property.

20. Derive the joint distribution of $X_{(r)}$ and $X_{(s)}$, the r^{th} and s^{th} order statistics.

($3 \times 8 = 24$ Marks)

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(Pages : 3) **P – 5257**

Reg. No. : .....

Name : .....

Second Semester M.Sc. Degree Examination, September 2022

Statistics

ST 522 — DISTRIBUTION THEORY

(2021 Admission)

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions. Each question carries **3** marks.

1. For any integer valued random variable X, show that

   $\sum_{n=0}^{\infty} S^n P(X \le n) = (1-s)^{-1} P(s)$, where $P(s)$ is the probability generating function of X.

2. Whether the marginal distributions uniquely determine the joint distribution? Justify.

3. If X and Y are independent binomial random variable such that $X \sim B(m, p)$ and $Y \sim B(n, p)$, show that $\frac{X}{X+Y}$ is hypergeometric.

4. State and prove one characteristic property of geometric distribution.

5. Let X have a (standard) Cauchy distribution. Find the probability density function of X^2. Identify its distribution.

P.T.O.

6. If and X and Y are independent r.v s with common $N(0, 1)$ distribution. Then $X+Y$ and X – Y are independent.

7. If $F(n_1, n_2)$ represents an F variate with (n_2, n_2) degrees of freedom, prove that $F(n_2, n_1) \sim \frac{1}{F(n_1, n_2)}$. Deduce that $P(F(n_2, n_1) \geq C) = P(F(n_1, n_2) \leq 1/c)$.

8. Define Non central t- distribution. When will this reduces to central-t distribution.

(4 × 3 = 12 Marks)

PART – B

Answer **any three** questions. Each question carries **8** marks.

9. Let $X \sim B(n, p)$. Find the mean and variance of Binomial distribution truncated at $X = 0$.

10. If X and Y are two random variables having joint density function

$$f(x, y) = \begin{cases} \frac{1}{8}(6 - x - y), 0 \leq x \leq 2, 2 \leq y \leq 4 \\ 0, \text{ otherwise} \end{cases}$$

Find (a) $P(X < 1 \cap Y < 3)$, (b) $P(X + Y < 3)$ and (c) $P(X < 1 | Y < 3)$.

11. For any two random variables X and Y, Show that $Var(X) = E(Var(X|Y)) + Var(E(X|Y))$.

12. If $X_1, X_2, .. X_k$ are k independent Poisson variates with parameters $\lambda_1, \lambda_2, .., \lambda_k$ respectively. Prove that the conditional distribution $P(X_1 \cap X_2, \cap ... \cap X_k | X)$, where $X = X_1 + X_2 + ... + X_k$ fixed is multinomial.

13. Define generalized power series distribution. Give any two particular cases of generalized power series distribution(g.p.s.d.). Also derive the recurrence relation for cumulants.

14. Define the negative binomial distribution. Show that under conditions the Negative binomial distribution tends to the Poisson distribution.

(3 × 8 = 24 Marks)

PART – C

Answer **any three** questions. Each question carries **8** marks.

15. Explain Weibull distribution. Show that $Min(X_1, X_2,..., X_n)$ follows Weibull if and only $X_1^1 s$ follows Weibull distribution.

16. X and Y are independent with a common pdf (exponential) $f(x)=e^{-x}, x \geq 0$
$= 0, x < 0$

    Find the distribution of X-Y. Identify the distribution.

17. Show that for a Pearsonian distribution specified by $\frac{1}{p}\frac{dp}{dx} = -\frac{a+x}{c_0+c_1x+c_2x^2}$, types are determined by the nature of roots of $c_0+c_1x+c_2x^2=0$.

18. Derive the sampling distribution of the sample mean and the sample variance of a random sample taken from a normal population with mean μ and variance σ^2.

19. (a) Find the pdf of $X_{(r)}$ in a random sample of size n from the exponential distribution $f(x) = \alpha e^{-ax}$, $\alpha \geq 0, x \geq 0$.

    (b) Show that $X_{(r)}$ and $W_{rs} = X_{(s)} - X_{(r)}$, $r < s$ are independently distributed.

    (c) What is the distribution of $X_{(r+1)} - X_{(r)}$.

20. Derive the moment generating function on non central Chi square distribution.

(3×8 = 24 Marks)

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