

Reg. No. : .....

Name : .....

First Semester M.Sc. Degree Examination, January 2026

Statistics

ST 512 : ANALYTICAL TOOLS FOR STATISTICS — II

(2021 Admission onwards)

Time : 3 Hours

Max. Marks : 60

PART – A

Answer any **four** questions. **Each** question carries **3** marks.

1. Define basis of a vector space. Give an example of an infinite dimensional vector space.
2. Show that an orthonormal set of vectors is linearly independent.
3. Define the column null space and row null space of a matrix.
4. Show that  $\rho(AA') = \rho(A'A)$ .
5. Show that the characteristic roots of an idempotent matrix are either 0 or 1.
6. Define algebraic multiplicity and geometric multiplicity of a characteristic root.
7. Define a quadratic form. What are different types of quadratic forms?
8. Show that  $\rho(\bar{A}A) = \rho(A)$ , where  $\bar{A}$  is a g-inverse of  $A$ .

(4 × 3 = 12 Marks)

PART – B

Answer any **three** questions. **Each** question carries **8** marks.

9. Define linear independence of vectors. Show that a set of non-zero vectors is linearly dependent if and only if a member in the set is a linear combination of its preceding vectors.
10. Show that an orthonormal basis always exists. Construct an orthonormal basis from  $\{(1, 1, 1), (-1, 0, -1), (-1, 2, 3)\}$ .
11. Define dimension of a vector space. If  $W_1$  and  $W_2$  are finite dimensional subspaces of a vector space  $V$ , then show that  $\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$ .

P.T.O.



12. If  $A$  and  $B$  are two square matrices of order  $n$ , then show that  

$$\rho(A) + \rho(B) - n \leq \rho(AB) \leq \text{Min}\{\rho(A), \rho(B)\}$$
13. State and prove rank-nullity theorem.
14. Derive the inverse of the partitioned matrix  $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ , where  $|A_{11}| \neq 0$ .

(3 × 8 = 24 Marks)

PART – C

Answer any **three** questions. **Each** question carries **8** marks.

15. Define characteristic roots and vectors. Show that the characteristic vectors corresponding to distinct characteristic roots of a matrix are linearly independent.
16. Compute  $A^{-1}$  and  $A^6$  using the Cayley-Hamilton theorem, where  $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$ .
17. Explain the spectral decomposition of a real symmetric matrix and discuss its properties and uses.
18. Derive the necessary and sufficient condition for the negative definiteness of a quadratic form.
19. Define g-inverse of a matrix and show that it is not unique. Also, show that a matrix  $\bar{A}$  is a g-inverse of  $A$  if and only if  $A\bar{A}A = A$ .
20. State and prove a necessary and sufficient condition for the system  $AX = B$  to be consistent. Find the value of  $\delta$  for which the following system of equations admit a solution :  $2x - y + 5z = 4, 4x + 6z = 1, -2y + 4z = 7 + \delta$ .

(3 × 8 = 24 Marks)



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U – 6594

Reg. No. : .....

Name : .....

**First Semester M.Sc. Degree Examination, February 2025**

**Statistics**

**ST 512 : ANALYTICAL TOOLS FOR STATISTICS - II**

**(2021 Admission Onwards)**

Time : 3 Hours

Max. Marks : 60

**PART – A**

Answer any **four** questions. Each question carries **3** marks.

1. Define vector space. Give an example.
2. Examine whether the following vectors are linearly independent in  $R^3$   $(1,1,1)$ ,  $(1,2,0)$  and  $(0,-1,2)$ .
3. Define a quotient space.
4. What do you mean by nilpotent matrix?
5. Explain eigen values and eigen vectors.
6. If A is a non-singular idempotent matrix then obtain its determinant.
7. What is Jordan Canonical form of a matrix?
8. Define negative definite matrix. Give an example.

**(4× 3 = 12 Marks)**

P.T.O.



PART – B

Answer any **three** questions. Each question carries **8** marks.

9. Distinguish between direct sum and complement of a subspace.
10. Show that two vectors are linearly independent if and only if one should be a scalar multiple of another.
11. Let  $V$  be a finite dimensional vector space. Show that all bases of  $V$  have same number of elements.
12. Explain Gram-Schmidt orthogonalization process through an example.
13. Show that every matrix is similar to a matrix of the form  $D + N$ , where  $D$  is diagonal and  $N$  is nilpotent.
14. Define rank of a matrix. Show that  $R(AB) \leq \min(R(A), R(B))$ .

**(3 × 8 = 24 Marks)**

PART – C

Answer any **three** questions. Each question carries **8** marks.

15. State and prove Cayley-Hamilton theorem.
16. (a) Define quadratic forms, illustrate different forms of them.  
(b) Find Moore-Penrose inverse of  $\begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$
17. Obtain a sufficient condition for a system of 'n' non-homogenous equations in 'n' unknown to yield a unique solution.
18. State and prove spectral decomposition theorem for real symmetric matrices.
19. Prove or disprove that the matrices  $AB$  and  $BA$  have same characteristics roots.
20. Prove that  $A^-$  is a g-inverse of  $A$  if and only if,  $AA^-A = A$ .

**(3 × 8 = 24 Marks)**



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S – 6272

Reg. No.

Name :

**First Semester M.Sc. Degree Examination, April 2024**

**Statistics**

**ST 512 : ANALYTICAL TOOLS FOR STATISTICS – II**

**(2021 Admission Onwards)**

Time : 3 Hours

Max. Marks : 60

PART – A

Answer **any four** questions. Each carries **3** marks

1. Define :
  - (a) vector space and
  - (b) subspace.
- ~~2.~~ Define Hermitian and skew Hermitian matrices.
3. Define linear independence.
4. What are orthogonal and unitary matrices?
5. What do you mean by inner product space?
6. Define algebraic and geometric multiplicities.
7. Explain different types of quadratic forms.
8. State the properties of g-inverse.

**(4 × 3 = 12 Marks)**

P.T.O.



PART – B

Answer **any three** questions. Each carries **8** marks.

9. Prove that the intersection of any two subspaces  $S$  and  $T$  of a vector space  $V$  is also a subspace of  $V$ .
10. Show that the vector space of all symmetric  $n \times n$  matrices with real entries has dimension  $\frac{n(n+1)}{2}$  as a vector space over  $R$ .
11. Explain Gram-Schmidt orthogonalization process through an example.
12. Define rank of a matrix. If  $A$  and  $B$  are square matrices of order  $n$ . show that  $r(AB) \geq r(A) + r(B) - n$ , where  $r(A)$  denotes the rank of  $A$ .
13. State and prove rank nullity theorem.
14. Explain the Spectral representation of real symmetric matrices.

**(3 × 8 = 24 Marks)**

PART – C

Answer **any three** questions. Each carries **8** marks.

15. Establish the relation between algebraic multiplicity and geometric multiplicity.
16. State and prove Cayley-Hamilton theorem.
17. Define Moore-Penrose generalized inverse of a matrix  $A$  and show that it is unique.



18. (a) Explain Jordan canonical form

(b) Find Jordan Canonical form of  $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ .

19. State and prove the expressions for trace and determinant of a square matrix in terms of its characteristic roots/eigen values.

20. Describe Gauss-Seidel method of solving a system of linear equation. Illustrate it by an example.

(3 × 8 = 24 Marks)

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R – 6197

Reg. No. : .....

Name : .....

**First Semester M.Sc. Degree Examination, May 2023**

**Statistics**

**ST 512 : ANALYTICAL TOOLS FOR STATISTICS - II**

**(2021 Admission Onwards)**

Time : 3 Hours

Max. Marks : 60

PART – A

Answer any **four** questions. **Each** carries **3** marks.

1. What is inner product of a vector space  $V$ ?
2. Show that the intersection of any number of subspaces of vector space  $V$  is a subspace of  $V$
3. Show that the only possible eigen values of an idempotent matrix are zero and one.
4. Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be defined by  $T(x,y) = (2x+3y, 4x-5y)$ . Find the matrix representation of  $T$  relative to the basis  $S = [u_1, u_2] = [(1,-2), (2,-5)]$ .
5. Show that the matrix  $A$  and its transpose  $A^T$  have the same characteristic polynomial.
6. Define algebraic and geometric multiplicities.

P.T.O.



7. Discuss the nature of the characteristic roots of a positive semi definite quadratic form.

8. Find the  $g$  – inverse of  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ .

(4 × 3 = 12 Marks)

PART – B

Answer any **three** questions. **Each** carries **8** marks.

9. Explain the Gram – Schmidt orthogonalization process by an example.

10. Let  $V$  be a vector space over the field  $F$ .  $U$  and  $W$  are subspaces of  $V$ . Show that  $\dim(U + W) = \dim U + \dim W - \dim(U \cap W)$ .

11. Explain linear dependence and linear independence. Determine whether or not the vectors  $u = (1,1,2)$ ,  $v = (2,3,1)$  and  $w = (4,5,5)$  in  $\mathbb{R}^3$  are linearly dependent.

12. Let  $U$  and  $W$  be the vector space over the same field  $F$  and let  $T$  be linear transformation from  $V$  into  $W$ . Suppose that  $V$  is finite dimensional. Prove that  $\text{rank}(T) + \text{nullity}(T) = \dim V$ .

13. Find the rank of the matrix  $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$

14. Define Hermitian matrix. If  $A$  is hermitian, then show that the eigen values of  $A$  are all real numbers.

(3 × 8 = 24 Marks)

PART – C

Answer any **three** questions. **Each** carries **8** marks.

15. Establish the relation between algebraic and geometric multiplicities of a characteristic root.

16. Obtain the characteristic roots and hence the characteristic vectors of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

17. Explain the spectral decomposition of a matrix. Find the spectral decomposition

of  $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

18. Reduce the quadratic form  $5x^2 + y^2 + 10z^2 - 4yz - 10zx$  to its canonical form and discuss its definiteness.

19. Use matrices to find the solution of

$$2x + y + z = 1$$

$$x - 2y - 3z = 1$$

$$3x + 2y + 4z = 5$$

20. Define Moore Penrose g – inverse and show that it is unique.

**(3 × 8 = 24 Marks)**

Reg. No. : .....

Name : .....

**First Semester M.Sc. Degree Examination, May 2022**

**Statistics**

**ST 512 : ANALYTICAL TOOLS FOR STATISTICS – II**

**(2021 Admission)**

Time : 3 Hours

Max. Marks : 60

**PART – A**

Answer any **four** questions. Each carries **3** marks.

1. Define subspace of a vector space. Prove that  $S = \{(a, b, c) \mid c \text{ is an integer}\}$  is not a subspace of  $\mathbb{R}^3$ .
2. Show that the vectors  $\alpha = (-2, 1)$  and  $\beta = (1, 2)$  are linearly independent.
3. Explain Idempotent matrix and Nilpotent matrix with an example of each.
4. Define similar matrices.
5. Explain spectral decomposition of a symmetric matrix.
6. Show that the modulus of each characteristic root of a unitary matrix is unity.
7. Define g-inverse of a matrix.
8. For which value of  $\mu$ , the following system of equations is inconsistent.  
 $3x - 2y + z = 10, 2x + 3y + 2z = 10, x + 2y + \mu z = 10.$

**(4 × 3 = 12 Marks)**

P.T.O.

PART – B

Answer any **three** questions. Each carries **8** marks.

9. Define basis and dimension of a vector space. Find the dimension and a basis of the solution space of the following homogeneous system.

$$x + 2y + z - 2t = 0$$

$$2x + 4y + 4z - 3t = 0$$

$$3x + 6y + 7z - 4t = 0$$

10. Using Gram – Schmidt orthogonalization process, construct an orthonormal basis from  $u_1 = (1, 1, 1, 1)$ ,  $u_2 = (1, 2, 4, 5)$  and  $u_3 = (1, -3, -4, -2)$ .

11. Define linear independence and linear dependence. Suppose two or more non zero vectors  $v_1, v_2, \dots, v_m$  are linearly dependent, Then show that one of them is a linear combination of the preceding vectors.

12. A mapping  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  is defined by

$T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2x_1 + x_2 + 2x_3, x_1 + 2x_2 + x_3)$ . Show that  $T$  is a linear mapping. Find the dimension of kernel and image of  $T$ .

13. For two matrices  $A$  and  $B$  of same order, show that  $\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B)$ .

14. Define symmetric matrix. If  $A$  and  $B$  are symmetric matrices, show that  $AB + BA$  is symmetric matrix.

(3 × 8 = 24 Marks)

PART – C

Answer any **three** questions. Each carries **8** marks.

15. Define algebraic and geometric multiplicity of a characteristic root and establish the relation between them.

16. Obtain the characteristic roots and hence the characteristic vectors of the matrix

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

*Handwritten notes:*  
 $(2, 2, 3)$   
 $(5, 2, 5)$   
 $(1, 1, 2)$

17. State and prove Cayley – Hamilton theorem.
18. Classify the following quadratic form as positive definite, positive semi-definite and indefinite  $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ .
19. Describe iterative method of solving a system of linear equation by an example.
20. Explain Moore Penrose inverse of a matrix. Find Moore Penrose inverse of
- $$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

**(3 × 8 = 24 Marks)**