(Pages : 2) **W – 5614**

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, January 2026

Statistics

ST 511 : ANALYTICAL TOOLS FOR STATISTICS – I

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer any **four** questions. Each question carries **3** marks.

1. Define an open ball in R^n.
2. What is a connected set? Give an example.
3. When do you say a function of bounded variation is Reimann integrable?
4. Establish the properties of Riemann-Stieltjes integrals.
5. Check whether $f(x,y) = x^2 + y^2 - 2ixy$ is an analytic function or not.
6. Use Cauchy's integral formula to evaluate $\int_x \frac{2+z^2}{z-1} dz$, where x is a circle $|z| = 2$
7. Prove that any zero of an analytic function is isolated in the set of its zeros.
8. Distinguish between poles and singularities in complex analysis.

(4 × 3 = 12 Marks)

SECTION – B

Answer any **three** questions. Each carries **8** marks.

9. (a) When do you say a metric space has the Bolzano-Weierstrass property?

   (b) State and prove the Bolzano-Weierstrass Theorem.

10. Show that a function is continuous if and only if the inverse image of every open set is open.

11. Define functions of bounded variation. If f and g are of bounded variation, then show that $f + g$ is also of bounded variation.

P.T.O.

12. State and prove the theorem establishing that a Riemann-Stieltjes integrable function reduces to a Riemann integrable function.

13. State and prove the two parts of the fundamental theorem of Integral Calculus.

14. Explain the procedure of finding maximum and minimum of functions of several variables using the Lagrangian multiplier method.

(3 × 8 = 24 Marks)

SECTION – C

Answer any **three** questions. Each carries **8** marks.

15. Let u and v be real valued functions defined on the domain $G \subset \mathbb{C}$ and suppose that u and v have continuous partial derivatives, then prove that $f: G \to \mathbb{C}$, defined by $f(z) = u(z) + iv(z)$ is analytic if and only if u and v satisfy the Cauchy-Reimann equations.

16. Let $f(z) = \begin{cases} \dfrac{x^3 - y^3 + i\left(x^3 + y^3\right)}{x^2 + y^2}, & \text{if } x \neq 0, y \neq 0 \\ 0, & \text{if } x = 0, y = 0 \end{cases}$. Then prove that the C-R equations are satisfied at the origin but $f(z)$ is not differentiable at the origin.

17. State and prove the maximum modulus principle.

18. Use Cauchy's Integral Formula or theorem to evaluate $\int_x \dfrac{\sin \pi z + \cos \pi z}{(z-1)(z-2)} dz$ where x is the circle $|z| = \dfrac{3}{2}$.

19. Discuss Cauchy's residue theorem. Using it compute $\int_C \dfrac{3x^2 + 4z^2 - 5z + 1}{(z-2i)(z^3 - z)} dz$, where $C: |z| = 3$ is the circle of radius 3 centered at 0.

20. Explain the various types of singularities

(3 × 8 = 24 Marks)

(Pages : 3) **U – 6593**

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, February 2025

Statistics

ST 511 : ANALYTICAL TOOLS FOR STATISTICS – I

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 60

PART – A

Answer any **four** questions. Each carries **3** marks.

1. If f is a real valued continuous function defined over n-dimensional Euclidean space. Show that $|f|$ is continuous.

2. Define Cauchy sequence. Give example.

3. State fundamental theorem of integral calculus.

4. Show that a constant function on $[a, b]$ is Reimann integrable.

5. State Cauchy's integral formula.

6. Define an analytic function and state the necessary condition for a function to be analytic.

7. Distinguish between residue at a pole and residue at infinity.

8. Define removable singularity of a function and give an example.

(4 × 3 = 12 Marks)

P.T.O.

PART – B

Answer any **three** questions. Each carries **8** marks.

9. State and prove Bolzano – Weierstrass theorem.

10. Prove that a function is continuous in a metric space if and only if the inverse image of every open set is open.

11. Give the Cauchy condition for uniform convergence. Let $\{f_n\}$ be a sequence of functions defined on a set S. Then show that there exist a function f such that $f_n \to f$ uniformly on S if and only if the Cauchy condition is satisfied.

12. Explain the procedure of obtaining Lagrange multipliers.

13. State and prove the first mean value theorem of Reimann Stieltjes integral.

14. Explain the Reimann Stieltjes integral. Explain its properties.

(3 × 8 = 24 Marks)

PART – C

Answer any **three** questions. Each carries **8** marks.

15. State and prove Liouvillie's theorem.

16. Find the analytic function whose real part is $\dfrac{x}{x^2+y^2}$.

17. State and prove Cauchy's integral theorem.

18. (a) State Cauchy' s Residue theorem.

    (b) Evaluate $\int_C \dfrac{3z-1}{z^3-z}dz$ where C is the circle

    (i) $|Z| = \dfrac{1}{2}$ and

    (ii) $|Z| = 2$.

19. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$ using Contour integration.

20. Name the singularities for the following functions.

(a) $\frac{1}{\cos\left(\frac{1}{z}\right)}$ at $z=0$

(b) e^{-1/z^2}.

(3 × 8 = 24 Marks)

(Pages : 3) **S – 6271**

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, April 2024

Statistics

ST 511 : ANALYTICAL TOOLS FOR STATISTICS — I

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions. Each carries **3** marks.

1. Show that $f+g$ is continuous if the function f and g are continuous.

2. Define metric space. Give example.

3. Show that the function $f(x, y) = x^2 - 2xy + y^2 + x^4 + y^4$ has minimum at the origin.

4. Define Reimann — Stieiltjes integral.

5. Is $f(z) = z^3$ analytic?

6. Evaluate $\int_c \frac{dz}{z^2(z-3)}$ on $|z| = 2$.

7. Define the terms pole and essential singularities giving one example each.

8. State Cauchy's Residue theorem.

(4 × 3 = 12 Marks)

P.T.O.

PART – B

Answer **any three** questions. Each carries **8** marks.

9. Define Cauchy sequence. Show that in Euclidean space R^k, every Cauchy sequence is convergent.

10. State and prove Heine – Borel theorem.

11. Define uniform continuity. If $f(x)=x^2$ for x in R, prove that f is not uniformly continuous on R.

12. State and prove fundamental theorem of integral calculus.

13. Find the maximum and minimum values of $x^2+y^2+z^2$ subject to the conditions $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{25}=1$ and $z=x+y$.

14. Show by an example that every bounded function need not be Reimann integrable.

(3 × 8 = 24 Marks)

PART – C

Answer **any three** questions. Each carries **8** marks.

15. State and prove Cauchy's Integral formula.

16. Find the analytical function $w=u+iv$, if $u=e^x(x\cos y-y\sin y)$. Hence find the harmonic conjugate v.

17. State and prove maximum modulus principle.

18. Discuss the singularities of the following functions

(a) $f(z)=\dfrac{z}{z^2+4}$

(b) $f(z)=\dfrac{e^{-z}}{(z-3)^4}$

19. Show that if an analytical function $f(z)$ has a pole of order m at $z=z_0$, then $\dfrac{1}{f(z)}$ has a zero of order m at $z=z_0$ and conversely.

20. Evaluate $\int_{-\infty}^{\infty} \dfrac{dx}{(x^2+1)^2}$ using contour integration.

(3 × 8 = 24 Marks)

(Pages : 2) **R – 6196**

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, May 2023

Statistics

ST 511 : ANALYTICAL TOOLS FOR STATISTICS – I

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 60

PART – A

Answer any **four** questions. Each question carries **3** marks.

1. Define Metric space.

2. Show that the union of an arbitrary family of open sets is open.

3. Define Riemann-Stieiltjes integral.

4. Show that the function $(y-x)^4+(x-2)^4$, has a minimum at (2, 2).

5. State Heine-Borel theorem.

6. Prove that the function e^z has an isolated essential singularity at $z=\infty$.

7. Find zeros and poles of $\left(\dfrac{z+1}{z^2+1}\right)^2$.

8. Define Cauchy residue theorem. Give one application.

(4 × 3 = 12 Marks)

P.T.O.

PART – B

Answer any **three** questions. Each question carries **8** marks.

9. Show that every open set is a union of open intervals.

10. Show that a metric space is discrete if and only if every point of the space is isolated.

11. Show that the function $f(x, y) = 2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum at (0,0), where $f_{xx}f_{yy} - (f_{xy})^2 = 0$.

12. Explain the Lagrange's method of multipliers.

13. State and prove fundamental theorem of integral calculus.

14. If $xyz = a^2(x + y + z)$, show that the minimum value of $yz + zx + xy$ is of $9a^2$.

(3 × 8 = 24 Marks)

PART – C

Answer any **three** questions. Each question carries **8** marks.

15. Prove that $u = y^3 - 3x^2y$ is a harmonic function. Determine its harmonic conjugate and then find the corresponding analytic function $f(z)$ in terms of z.

16. State and prove Liouvilles theorem.

17. Show that the equation $z^4 + z + 1 = 0$, has one root in each quadrant.

18. Evaluate the integral $\int_0^{2\pi} \cos^{2n} \theta d\theta$, where n is a positive integer.

19. Evaluate by contour integration that: $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2} dx = \pi \log 2$.

20. State and prove the maximum modulus principle.

(3 × 8 = 24 Marks)

(Pages : 3) **N – 5386**

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, May 2022.

Statistics

ST 511 – ANALYTICAL TOOLS FOR STATISTICS – I

(2021 Admission)

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions. Each question carries **3** marks.

1. Define Cauchy sequence.

2. If $\{a_n\}$ and $\{b_n\}$ are bounded sequences, then prove that

   $\underline{\lim}(a_n + b_n) = \underline{\lim}(a_n) + \overline{\lim}(b_n)$

3. Show that every closed subset of a compact metric space is compact.

4. Show that $f(x,y) = y^2 + x^2y + x^4$, has a minimum at (0,0).

5. State Cauchy Integral formula.

6. Show that every convex domain is simply connected.

7. Define an analytic function. Give one example.

8. Define Cauchy residue theorem. Give one application.

(4 × 3 = 12 Marks)

P.T.O.

PART – B

Answer **any three** questions, Each question carries **8** marks

9. Prove that a metric space (X, d) is sequentially compact if and only if it has the Bolzano-weierstrass property.

10. Show that a metric space is discrete if and only if every point of the space is isolated.

11. If f_1 and f_2 are two bounded and integrable functions *on* $[a, b]$, then show that $f = f_1 + f_2$ is also integrable on $[a, b]$ and $\int_a^b f_1 dx + \int_a^b f_2 dx = \int_a^b f\, dx$.

12. Define Riemann Stieltjes integral. How it is related to the Riemann integral.

13. State and prove fundamental theorem of integral calculus.

14. Show that the function f defined by $f(x) = \begin{cases} 0, & when\ x\ is\ rational \\ 1, & when\ x\ is\ irrational \end{cases}$ is not integrable on any interval.

(3 $\times$ 8 = 24 Marks)

PART – C

Answer **any three** questions Each question carries **8** marks

15. Show that the function $v(x, y) = Cosx, Coshy$ is harmonic and find the corresponding analytic function.

16. State and prove Liouvilles Theorem.

17. Show that $\int_0^\infty \frac{(\log x)^2}{1+x^2} dx = \frac{\pi^2}{8}$.

18. State and prove the necessary and sufficient condition for a function to be analytic.

19. By contour integration prove that

$$\int_0^\infty \frac{\sin^2 x}{x^2}\,dx = \frac{\pi}{2}.$$

20. State and prove maximum modulus principle.

($3 \times 8 = 24$ Marks)

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